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| 1. Course title: Analysis in Several Variables lecture | | | | | |
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| 2. Code: | | 3. Type (lecture, practice etc.): lecture | | | |
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| 4. Contact hours: 3 hoursper week | | 5. Number of credits (ECTS): 3 | | | |
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| 6. Preliminary conditions (max. 3): Analysis 2 lecture+ seminar | | | | | |
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| 7. Announced: fall semester,  spring semester, both | | | | | |
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| 8. Limit for participants: 40 | | | | | |
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| 10. Responsible teacher (faculty, institute and department):  Margit Pap PhD (Faculty of Science, Institute of Mathematics and Informatics, Department of Mathematics) | | | | | |
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| 11. Teacher(s) and percentage: | | Dr. Margit Pap | | 100 % | |
| Dr. Tímea Eisner | | 100 % | |
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| 12. Language:English | | | | | |
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| 13. Course objectives and/or learning outcomes:  **Objectives**: The lecture intends to introduce students to the basic notions of Multivariable Analysis: concepts of limits, continuous functions, differentiability, **multivariable differential and integral calculus and their applications**. The course helps the development of problem solving skills.  Learning outcomes: students completing the course will have *knowledge* on basic concepts and theorems of Multivariable Analysis. They will be *able* to apply the properties of these concepts. They will have a *competence* of evaluating readings in Analysis. Their positive *attitude* towards methods calculating limits will increase significantly. | | | | | |
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| 14. Course outline   1. Metric space, norm space, Euclidean space. (Cauchy-Schwarz inequality, Minkowski-inequality with proof), Banach-space, Hilbert-space. Examples. Boundedness of sets in metric spaces and in **Rn**. 2. Boundedness and convergence of sequences in **Rk**. Uniqueness theorem, Theorem ont he connection of convergence and the convergence of components with proof.) 3. 3 consequences of convergence of sequences in **Rk** .The **Rk** space: Bolzano-Weierstrass selection theorem, Cauchy-test for convergence. Topological notions in metric spaces: open sets, theorem on open sets, closed sets, theorems on closed sets. Compact sets. Connections between compactness, boundedness, closeness. (with proof.) 4. Functions in metric spaces. Notion of multivariable functions. Notion of linear functions in linear space. Representation of linear functions in space **Rk**. Congestion point. Notion of function limit in metric spaces and in **Rk**. Its connection with limit of sequences. Limit properties. Theorem on the existence of iterated limit. (with proof) 5. Continuous functions in metric spaces and in **Rk**, its connection to sequences. Discontinuities. Operations (linear combinations, product, quotient, composition) Properties of continuous functions on compact sets. (3 theorems) Connected sets. Generalisation of theorem of Bolzano. (Topology, dimension, fractals.) 6. Differentiability of multivariable functions. Partial derivatives, directional derivatives. Geometrical meaning. Connection between partial differentiability and continuity, with proof. 3 equivalent notions of differentiability, Derivative matrix, gradient, geometrical meaning, Jacobean matrix. Tangent plane. 7. Connection between differentiability and continuity. (With proof.) A differentiable function is partially differentiable with respect to each variables. Counterexample for the failure of the reverse statement. A sufficient condition of the differentiability. (Proof for 2 variables.) 8. Derivatives of function composition. Generalizations of the Lagrange mean value theorem. (With proof.) Higher order derivatives. First and second order differential. Young theorem on the equality of the mixed partial derivatives. (With proof.) 9. Bivariate Taylor formula. (With proof.) Local extrema for real functions of several variables. A first order necessary condition. The quadratic form of a symmetric matrix. Theorems on definite matrices. Second order sufficient condition on the existence of extrema. Second order necessary condition on the existence of constrained extrema. 10. Implicit function theorem for one-variable function equations. Constrained extrema. The necessary condition on the existence of constrained extrema. Lagrange multiplier method. 11. Double integral. Operation properties. Fubini theorem. Evaluation of double integrals on bounded regions. Multiple integrals, a generalisation in *n* dimensions. Generalisations of theorems on one-variable integral calculus. 12. Interal transformation. Polar coordinate transformation. Computing volumes. 13. Jordan measure. Applying the double integral to compute area, volume, mass, centres of gravity. Evaluation of a Gauss integral applying a double integral. | | | | | |
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| 15. Mid-semester works  Attending lectures is compulsory. | | | | | |
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| 16. Course requirements and grading  Written exam is based on lectures, accessible electronic sources and lecture materials.  There is a written preliminary exam. Preliminary exam grades:  0–54% fail  55–69% acceptable  70–79% average  80–89% good  90–100% excellent  After successful preliminary exam there is an oral exam in 3 topics. The final grade is obtained from the arithmetic mean of the 4 grades, but only in case when all parts hit the acceptable measure. | | | | | |
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| 17. List of readings  Rudin, Walter. Principles of mathematical analysis. Vol. 3. New York: McGraw-Hill, 1964.  Stewart, James. Calculus: early transcendentals. Cengage Learning, 2015.  Dineen, Seán, Multivariate calculus and geometry. Springer, 2001.  Moskowitz, Martin A., and Fotios Paliogiannis. Functions of several real variables. World Scientific, 2011. | | | | | |
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| 18. Recommended texts, further readings  Joel R. Hass, Christopher D. Heil, Maurice D. Weir. Thomas' Calculus, 14th Edition. | | | | | |
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| **Date** | 14 May, 2017 | **Prepared by** |  | | |
| **Dr. Margit PAP** responsible teacher | | |
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| **Endorsed by** | | |  | | |
| Dr. László TÓTH program supervisor | | |